

Rotational Analog of the Hall Effect: Coriolis Contribution to Electric Current

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A galvanogyroscopic effect which is the rotational analog of the gravitomagnetic Hall effect has been proposed. As a consequence of Ohm's law in the rotating frame, the effect of the Coriolis force on the conduction current is predicted to give rise to an azimuthal potential difference V_{gg} about $10^{-3}V$ in a spinning rotor carrying radial electric current i_r . The potential difference developed by the galvanogyroscopic effect is proportional both to angular velocity Ω and to the electric current. Key words: Ohm's law, galvanogyroscopic effect, rotation, Coriolis force.

Our aim here is to incorporate a Coriolis force term to the theory of electrical conductivity, which could lead to a galvanogyroscopic effect. Then the predicted effect is applied to design a verifiable experiment to detect an influence of the rotation of the earth on an electric current flowing in a conductor.

The point is that in the rotating frame of reference conduction electrons are affected by the Coriolis force

$$\mathbf{F} = 2m[\mathbf{v} \times \boldsymbol{\Omega}] , \quad (1)$$

\mathbf{v} is the velocity of the conduction electrons, m is the electron mass and $\boldsymbol{\Omega}$ is the angular velocity of rotation.

According to the Larmor's theorem the Coriolis force imitates the equivalent magnetic field \mathbf{B}_{eq} applied to the immovable medium:

$$\mathbf{F} = -(|e|/c)[\mathbf{v} \times \mathbf{B}_{eq}] , \quad \mathbf{B}_{eq} = -(2mc/|e|)\boldsymbol{\Omega} , \quad (2)$$

where $e = -|e|$ is the electronic charge and c is the velocity of light in vacuum. That is the effect of the Coriolis force on conduction electron is the same as that of the Lorentz force due to the equivalent magnetic field.

In the Hall effect⁽¹⁾, under influence of a magnetic field on conduction current, an electric field

$$\mathbf{E}_H = R_H \mathbf{j} \times \mathbf{B} \quad (3)$$

appears in a conductor to which an electric current \mathbf{j} and a magnetic field \mathbf{B} are applied in perpendicular directions, where R_H is the Hall constant. Consequently according to equations (2) and (3) there exists a galvanogyroscopic effect which can be summarized by saying that an inner electric field being perpendicular to the current and angular velocity is induced by rotation in a spinning crosslike rotor carrying a radial electric current i_r (see Figure). Then the voltmeter applied in an azimuthal direction would detect the galvanogyroscopic voltage V_{gg} . This effect has the following properties: (i) galvanogyroscopic effect is odd, that is, the galvanogyroscopic voltage changes sign in dependence of the polarity of

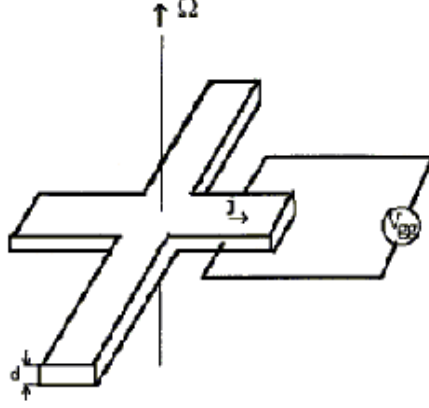


Figure 1: Crosslike rotor carrying radial electric current i_r where the galvanogyroscopic voltage V_{gg} is produced.

either the angular velocity or the electric current, (ii) in the first approximation, the voltage V_{gg} is proportional to Ω and described by the formula

$$V_{gg} = (R_{gg}/d)i_r\Omega , \quad (4)$$

where $R_{gg} = 2m/ne^2$ is the galvanogyroscopic coefficient, n is the electron concentration, and d is the thickness of the plate of the rotor.

The galvanogyroscopic effect may be regarded, among other things, as a consequence of Ohm's law in the rotating frame which can be obtained by extending the usual derivation of constitutive equations to include noninertial acceleration. The equation of motion for conduction electrons in rotating frame can be written as

$$m\frac{d\mathbf{v}}{dt} - m\mathbf{\Omega} \times (\mathbf{r} \times \mathbf{\Omega}) - 2m\mathbf{\Omega} = -e(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}) + \frac{ne^2}{\sigma}\mathbf{v} , \quad (5)$$

where the last term is due to the resistance force acting on conduction electrons from continuous medium with conductivity σ , \mathbf{E} is the electric field. It is clear that the equations (5) would involve the effective mass of the conduction electrons, rather than the free mass. However as it was discussed and demonstrated by many authors (see⁽²⁻⁴⁾ for review and most recent results) that the use of the free mass of the electron instead of the effective one is very good approximation on measurement of electromotive force in accelerated conductors.

Taking into account the fact that the steady state $\frac{d\mathbf{v}}{dt} = 0$ establishes within the conductor in extremely short time ($\tau \approx 10^{-13}s$), one arrives at Ohm's law for the current density $\mathbf{j} = ne\mathbf{v}$ which flows in the rotating conductor

$$\mathbf{E} = \frac{1}{\sigma}\mathbf{j} + R_H\mathbf{j} \times \mathbf{B} - R_{gg}\mathbf{j} \times \mathbf{\Omega} + \frac{m - \gamma M_a}{e}\mathbf{\Omega} \times (\mathbf{r} \times \mathbf{\Omega}) . \quad (6)$$

Except for ordinary standard terms, the current flowing in the rotating conductor has two contributions: one is due to the Coriolis force effect on current which is represented by the term $R_{gg}\mathbf{j} \times \mathbf{\Omega}$ and other one is due to the centrifugal force. The correction γM_a (γ is the parameter of order 1 and M_a is the atomic mass) in the right hand side of the equation (6) is due to the strain gradients produced by the acceleration field (see, for example,⁽⁵⁾).

Ohm's law for conduction current has, moreover, been generalized to include effects of gravity and inertia in recent papers⁽⁴⁻¹⁰⁾ where, in particular, the electromagnetic fields arising from the centrifugal acceleration in the rotating conductors are calculated. However the effect of the Coriolis force upon an electric current has been taken into account only in⁽⁹⁾ where the experimentally measured magnetic field⁽¹⁰⁾ around a rotating conductor was explained as a result of azimuthal current produced by the effect of Coriolis force on radial thermoelectric current. Furthermore it has been experimentally established that centrifugal forces produce a potential drop in a spinning conductor^(4,12) (see⁽³⁾ for review) and consequently act on the electrons within accelerating conductors as a true electric field applied to the conductor.

Consider now a crosslike rotor carrying radial current and spinning with a high angular velocity as in Figure. Suppose that the external magnetic field is absent. Charges moving radially inward or outward will experience a Coriolis force¹, which will cause some motion of charges relative to the lattice in the azimuthal direction. This motion will be compensated by an azimuthal electric field arising from charge separation on opposite sides of rotor. As we can see from Ohm's law (6), the effect of the Coriolis force on the current gives rise to an electric field perpendicular to the current, whose magnitude is defined by (4).

The potential difference produced should be minute in metals, but appreciable in semi-

¹The centrifugal force being proportional to Ω^2 has been neglected in the linear approximation in the angular velocity of the rotation $\mathbf{\Omega}$, which is hereafter used for simplicity of calculations.

conductors because the concentration of electric carriers $n \approx 10^{-14}cm^3$ much smaller in semiconductors in comparison to that in conductors. The sign (+ or -) of the coefficient $R_{gg} \approx 0.8 \times 10^{-22}s$ depends on the electric charge of the majority carriers, hence measurement of the R_{gg} coefficient should determine both the sign and the concentration of the majority carriers.

For typical values of parameters $\Omega = 10^3s^{-1}$, $i_r = 10^3A$ and $d = 10^{-2}cm$ in the absence of any additional effects one would expect that $V_{gg} = 0.7 \times 10^{-2}V$. An experiment to observe this voltage is to rotate a rotor carrying radial current and measure the potential difference through it in azimuthal direction (for example, radial current in the rotor can be supported by thermoelectric effects as in the experiment⁽¹¹⁾). The most dangerous sources of experimental error, arising from the earth's magnetic field, ionization in surrounding air etc. should be circumvented during the experiment with the required accuracy. Measuring the galvanogyroscopic potential difference as a function of conduction current thus yields a direct measure of rotation rate which, in principle, can be used for construction of new type devices such as high sensitivity speedometers of angular velocity.

If the semiconductor carrying radial current is at rest with respect to the earth, then there would be a voltage due to the earth's diurnal rotation with angular velocity $\Omega_{\oplus} \approx 7.3 \times 10^{-5}s^{-1}$. From real estimations, for parameters $i_r = 10^3A$ and $d = 10^{-2}cm$ one finds that an azimuthal voltage $V_{\oplus} \approx 0.5 \times 10^{-10}V$ will be produced by galvanogyroscopic effect across a semiconductor rigidly rotating together with the earth. The measurement of this voltage based on advanced experimental techniques⁽¹³⁾ should reprove the diurnal rotation of the earth, and in this sense, is analogous to the mechanical measurements made with the Foucault pendulum⁽¹⁴⁾ for observation of earth's rotation.

In this paper, we have shown that the effect of the Coriolis force on the conduction current is to induce a galvanogyroscopic potential difference. Thus electrodynamic analog of the well known mechanical Beer's law arising from the effect of the Coriolis force on the stream of a river has been found. Similarly, according to the galvanogyroscopic effect one lateral side of the rotating conductor with electric current has opposite electric charges relative to the other one due to the interaction between electric current and Coriolis force.

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